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Lumped damping and stability of Beck column with a tip mass

F.M. Detinko

2428 Lake Vista Ct., Apt 210, Casselberry, FL 32707, USA

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Abstract

Lumped external damping, acting on the concentrated mass at the tip of Beck's column, is taken into account. This additional external damping removes the prolonged inconsistency between theory and experiment for a column with a tip mass, subjected to follower force.

In Sugiyama et al. [J. Aerospace Eng. 8 (1995) 9] and Wood et al. [Proc. Roy. Soc. Lond. Ser. A. 313 (1969) 239] the experimental critical loads for a beam with a tip mass agree well with theoretical values for an undamped system. On the other hand, the critical loads, calculated in Sugiyama et al. with account for the internal and distributed external damping, were 50% lower than the experimental values. Since the actual experimental system is subject to damping, the experiment and theory come into conflict. This conflict is resolved in this paper by observing that the experimental system was subjected not only to the distributed external damping (due to distributed mass), but also to the lumped damping, acting on the tip mass. When the lumped external damping included in the analysis, analytical critical forces are in good agreement with experimental ones.

Keywords: Stability; Follower force; Experiment; Lumped damping

1. Introduction

Experimental investigation of the flutter instability of a cantilevered beam with a tip mass, subjected to a follower force, was described in Sugiyama et al. (1995) and Wood et al. (1969). In both experiments critical loads were in good agreement with those calculated for an undamped system, even though it is known that there is damping in practice.

In Sugiyama et al. the tangential load was realized by installation of a solid rocket motor directly on a cantilevered column. The experimental critical loads were slightly lower than calculated on the basis of no damping. When the distributed external and internal damping were accounted for, the theoretically predicted critical loads were about 50% of the measured value.

In Wood et al. the tangential load was obtained by a jet of water, issuing from a nozzle box. By changing the nozzle box mass, as well as the length and mass of a beam, a total of 36 experimental points were

E-mail address: felideti@k2services.com (F.M. Detinko).

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obtained. The discrepancy between measured critical loads and calculated for an undamped system did not exceed 3%.

The proximity of critical load, calculated for an undamped system, to test is inconsistent to the well known theoretical findings for Beck's column: the arbitrary small, but finite internal damping, which is always present in a real system, drastically reduces the follower critical load (Bolotin and Zhinzher, 1969). Only when the ratio of external damping to internal one tends to infinity, does the critical load asymptotically approach the value, calculated for an undamped Beck's column (Gajewski, 1972; Denisov and Novikov, 1975).

However, in both experimental setups there was a distinctive feature: concentrated mass at the tip of a beam and, therefore, a lumped external friction force acting on the mass. This additional damping is accounted for in this paper.

2. Analysis

The mathematical model of the column is shown in Fig. 1. To apply the follower force at the tip of the column, there is a concentrated mass M with a moment of inertia J, the center of gravity of which is located at a distance a from the end of a column. The Voigt–Kelvin material of the column with Young's modulus E and viscosity modulus E^* is assumed. Equation of small motion of the column is

$$EI\frac{\partial^4 y}{\partial x^4} + E^*I\frac{\partial^5 y}{\partial t\,\partial x^4} + P\frac{\partial^2 y}{\partial x^2} + K\frac{\partial y}{\partial t} + m\frac{\partial^2 y}{\partial t^2} = 0 \tag{1}$$

where m is mass per unit of length of column, I is its moment of inertia, and K is coefficient of external distributed damping.

Boundary conditions at the fixed end are

$$y = \frac{\partial y}{\partial x} = 0 \quad \text{at } x = 0 \tag{2}$$

At the loaded end $x = 1$

$$\begin{aligned} &EI\frac{\partial^2 y}{\partial x^2} + E^*I\frac{\partial^3 y}{\partial t\,\partial x^2} = -\frac{\partial^2}{\partial t^2}\left[J\frac{\partial y}{\partial x} + MaY\right] - \frac{\partial}{\partial t}\left[K_2\frac{\partial y}{\partial x} + K_1 aY\right] \\ &EI\frac{\partial^3 y}{\partial x^3} + E^*I\frac{\partial^4 y}{\partial t\,\partial x^3} = M\frac{\partial^2 Y}{\partial t^2} + K_1\frac{\partial Y}{\partial t}, \quad Y = y + a\frac{\partial y}{\partial x} \end{aligned} \tag{3}$$

The last term in the second equation represents the lumped damping force, applied at the center of gravity of the tip mass. The term proportional to K_2 is lumped damping moment due to rotary inertia of the tip mass. These forces were not accounted for in Sugiyama et al. (1995), Andersen and Thomsen (2002). In Wood et al. (1969) damping was ignored completely.

Fig. 1. Mathematical model.

Introducing the dimensionless quantities

$$\xi = x/L, \quad \alpha = a/L, \quad \mu = M/mL, \quad v = J/mL^3, \quad p = PL^2/EI$$

$$\tau = \frac{t}{L^2}\sqrt{EI/m}, \quad \gamma = \frac{E^*}{EL^2}\sqrt{EI/m}, \quad k = \frac{KL^2}{\sqrt{mEI}}, \quad k_1 = \frac{K_1 L}{\sqrt{mEI}}, \quad k_2 = \frac{K_2}{L\sqrt{mEI}}$$

and searching the solution of (1) in the form

$$y(x,t) = Lf(x)\exp(q\tau) \tag{4}$$

one obtains the equation and boundary conditions for $f(x)$ as

$$(1+\gamma q)f''''(\xi) + pf''(\xi) + (kq + q^2)f(\xi) = 0 \tag{5}$$

$$(1+\gamma q)f''(1) = -Q_2 f'(1) - \alpha Q_2[f(1) + \alpha f'(1)] \tag{6}$$

$$(1+\gamma q)f'''(1) = Q_1[f(1) + \alpha f'(1)] \tag{7}$$

$$f(0) = f'(0) = 0 \tag{8}$$

where $Q_1 = \mu q^2 + k_1 q$, $Q_2 = vq^2 + k_2 q$. The solution to (5) satisfying boundary condition (8) can be written as

$$\begin{aligned} f(\xi) &= A[\cosh(g_2\xi) - \cos(g_1\xi)] + B[g_1\sinh(g_2\xi) - g_2\sin(g_1\xi)] \\ g_{1,2}^2 &= \frac{\sqrt{p^2 - 4q(q+k)(1+\gamma q)} \pm p}{2(1+\gamma q)} \end{aligned} \tag{9}$$

where A, B are the integration constants. The boundary conditions (6) and (7) lead to the characteristic equation, the explicit form of which is given in Appendix A.

On the boundary of flutter instability the eigenvalue is a pure imaginary number $q = \mathrm{i}\omega$ and the characteristic equation takes the form

$$\Delta(p,\omega) = \mathrm{Re}(p,\omega) + \mathrm{i}\,\mathrm{Im}(p,\omega) = 0$$

The critical load and frequency of vibration at this load are found from two equations:

$$\mathrm{Re}(p,\omega) = 0 \tag{10}$$

$$\mathrm{Im}(p,\omega) = 0 \tag{11}$$

For the undamped system ($\gamma = k = k_1 = k_2 = 0$) Eq. (11) becomes an identity and the critical load is found from the condition that two first eigenvalues of (10) coalesce.

3. Experimental results and comparison to analysis

Properties of experimental setups are listed in Table 1.

3.1. *Sugiyama et al. (1995) test*

Using experimental critical loads P (Sugiyama et al., 1995, Table 3) dimensionless loads $p_t = PL^2/EI$ are calculated and listed in Table 2 for different lengths of a bar. The analytical results p_0, recalculated for undamped case ($\gamma = k = k_1 = k_2 = 0$), are also listed in Table 2. On the average, $p_t/p_0 = 0.97$.

Table 1
Properties of experimental setups

Property	Units	Sugiyama et al. (1995)	Wood et al. (1969)
Tip mass, M	kg	14.18	0.428
Mass moment of inertia, J	kg m^2	0.1196	7.67E−4
Distance to CG, a	m	0.20	0.0348
Mass per unit length, m	kg/m	0.481	0.0939
Bending stiffness, EI	N m^2	33.6	0.224
Column length, L	m	1.05	0.51

Table 2
Comparison of measured critical loads to analytical for undamped system in Sugiyama et al. (1995)

Length, m	Test, p_t	Analysis with no damping, p_0
1.10	12.0	12.92
1.05	12.5	12.76
1.025	12.2	12.68
1.00	12.5	12.59

As was proven by Namat-Nasser (1967), the flutter critical load of the system without damping is the upper bound for the same system with slight internal damping, that is the latter is lower or equal to the former, but cannot be higher. Experimental results in Table 2 suggest that this may be also true for the system with both internal and external damping.

It was shown (Gajewski, 1972; Denisov and Novikov, 1975) that the analytical critical load for a damped Back's column depends only on the ratio k/γ of distributed external to internal damping factors (when both are small), not on these two values separately. Our calculations show that this is also true for a column with a tip mass and lumped damping.

The reason for this is twofold. First, the roots of Eq. (10) are not affected by small damping factors. Secondly, Eq. (11) after linearization relative to damping factors is written as

$$\mathrm{Im}(p,\omega) = B_1(p,\omega)\gamma + B_2(p,\omega)k + B_3(p,\omega)k_1 + B_4(p,\omega)k_2 = 0$$

or, for $\gamma \neq 0$,

$$B_1(p,\omega) + B_2(p,\omega)\frac{k}{\gamma} + B_3(p,\omega)\frac{k_1}{\gamma} + B_4(p,\omega)\frac{k_2}{\gamma} = 0$$

That is why all analytical critical loads are given below as functions of damping factor ratios.

Obviously, it is impossible to calculate damping factors for the experimental setups. However, using various ratios k/γ, k_1/γ, k_2/γ we can find out how the critical load depends on them and whether it is possible to match experimental critical loads by calculations which include damping. Fig. 2 shows calculated critical loads for $k = 0$ and variable k_1/γ, k_2/γ; in Fig. 3 $k_2 = 0$ and k/γ, k_1/γ are variable.

Consider Fig. 2, where the ratio of critical loads p/p_0 is depicted vs. k_1/γ for $k = 0$ and $k_2 = 0$, 6γ, 20γ (curves 1, 2, 3 respectively). For $k_2 = 0$, 6γ (curves 1, 2) the ratio p/p_0 first increases with increased k_1/γ, reaches a maximum equal to unity, and then decreases. Hence, for low k_2/γ the damping k_1/γ may either stabilize or destabilize the column. At the maximum the critical load reaches its upper bound, and for a wide range of k_1/γ the calculated critical load p is bounded in the narrow interval:

$$p_0 \geqslant p \geqslant 0.97p_0 = p_t \tag{12}$$

(p_t is the measured critical load). When $k_2 \geqslant 20\gamma$ the translational lumped damping k_1 always stabilizes the column (see curve 3).

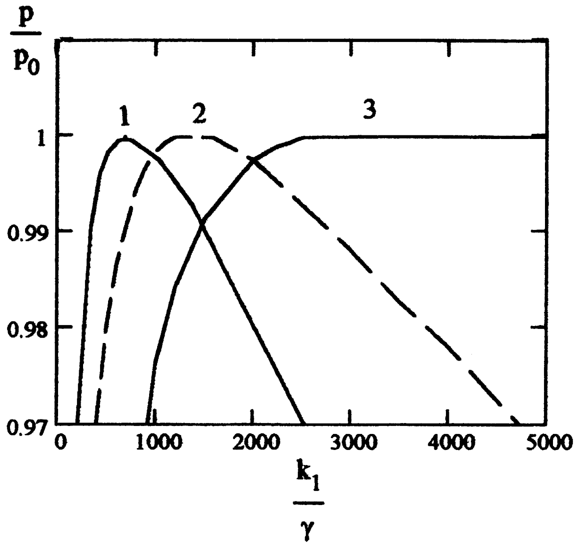

Fig. 2. Sugiyama et al. (1995) test. Critical load p/p_0 ($p_0 = 12.76$ with no damping) vs. lumped damping k_1/γ for $k = 0$. (1) $k_2 = 0$; (2) $k_2 = 6\gamma$; (3) $k_2 = 20\gamma$.

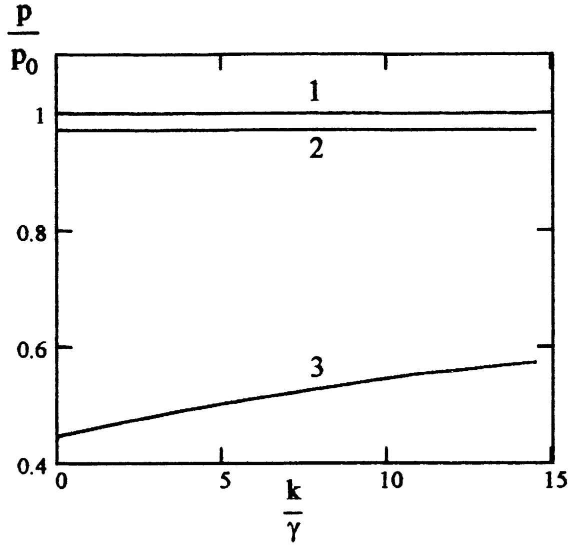

Fig. 3. Sugiyama et al. (1995) test. Critical load p/p_0 ($p_0 = 12.76$ with no damping) vs. distributed damping k/γ for $k_2 = 0$. (1) $k_1/\gamma = 689$; (2) $k_1/\gamma = 224$, 2550; (3) $k_1/\gamma = 0$. Between lines 1 and 2 $224\gamma \leqslant k_1 \leqslant 2550\gamma$.

In Fig. 3 the critical load is depicted vs. distributed damping k/γ for $k_2 = 0$ and different values of k_1/γ. Curve 3 shows the effect of internal and external distributed damping only. Point $k/\gamma = 2.4$ on this curve represents the critical load, calculated in Sugiyama et al. (1995). On line 1 $k_1/\gamma = 689$ (maximum point on curve 1 of Fig. 2); between lines 1 and 2

$$224 \leqslant k_1/\gamma \leqslant 2550 \tag{13}$$

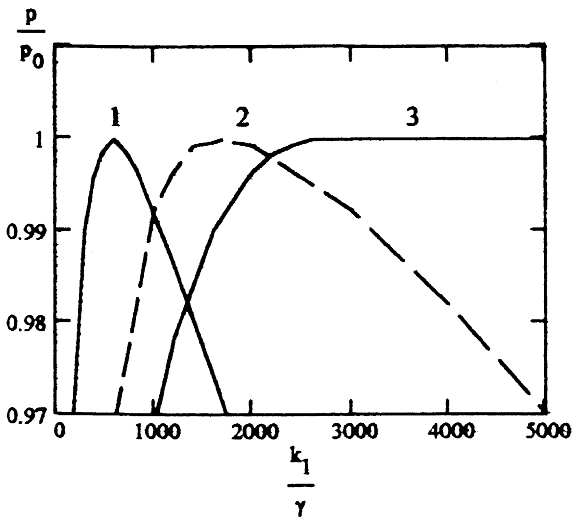

Fig. 4. Wood et al. (1969) test. Critical load p/p_0 ($p_0 = 14.35$ with no damping) vs. lumped damping k_1/γ for $k = 0$. (1) $k_2 = 0$; (2) $k_2 = 10\gamma$; (3) $k_2 = 20\gamma$.

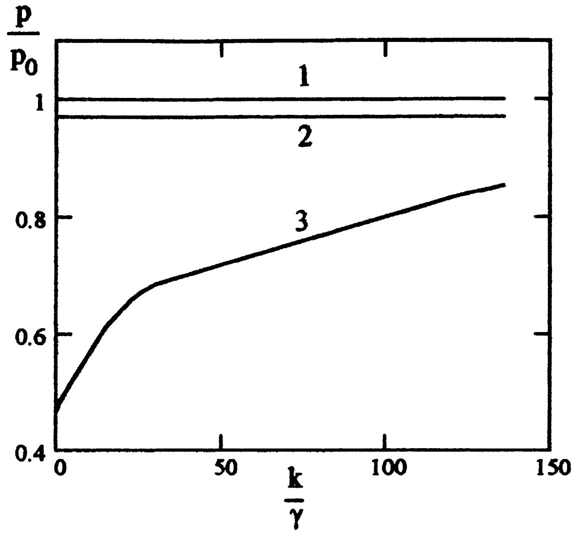

Fig. 5. Wood et al. (1969) test. Critical load p/p_0 ($p_0 = 14.35$ with no damping) vs. distributed damping k/γ for $k_2 = 0$. (1) $k_1/\gamma = 592$; (2) $k_1/\gamma = 178$, 1752; (3) $k_1/\gamma = 0$. Between lines 1 and 2 $178\gamma \leqslant k_1 \leqslant 1752\gamma$.

($p/p_0 = 0.97$ on curve 1 of Fig. 2). It is seen that for the whole interval (13) the critical load does not depend on k/γ. This means that when the tip mass is much higher than the distributed one (in this case $\mu = M/mL = 28$) the effect of distributed damping is negligible.

3.2. Wood et al. (1969) tests

The tests results in Wood et al. are given in the form of plots only and it is difficult to accurately read numerical values. The critical loads are equal or somewhat lower than loads, calculated for the undamped system and we will assume that on the average, the equality $p_t/p_0 = 0.97$ holds. Figs. 4 and 5 are similar to Figs. 2 and 3 respectively but both bounds of the interval (13) are lower, because the relative tip mass is smaller, $\mu = 8.9$.

4. Conclusion

The important conclusions are as follow:

1. For Beck's column with a tip mass the lumped damping at the end of the column should be taken into account.
2. At one particular (optimum) value of the lumped damping the critical load reaches its upper bound, equal to the critical load for an undamped system.
3. In a wide range around this optimum value the critical load is slightly lower than the upper bound.
4. In the experiments the destabilizing effect of internal damping was most likely masked by the lumped external damping due to wind resistance. To verify this statement by test it would be desirable to drastically reduce this damping. Wood et al. mentioned that "... a simpler system is being developed to investigate the effect" of "variable friction", but to the best of this writer's knowledge nothing of this sort was ever published.

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Appendix A

Roots g_1, g_2 are given by (9).

$$\Delta(p,\omega) = (1+\gamma q)^2 A_1 - (1+\gamma q)(A_2 + \alpha A_3 + \alpha^2 A_4)Q_1 - (1+\gamma q)Q_2 A_4 + Q_1 Q_2 A_5$$
$$q = \mathrm{i}\omega, \quad Q_1 = \mu q^2 + k_1 q, \quad Q_2 = v q^2 + k_2 q$$

$$A_1 = g_1 g_2[g_1^4 + g_2^4 + 2g_1^2 g_2^2 \cosh(g_2)\cos(g_1) + g_1 g_2(g_1^2 - g_2^2)\sinh(g_2)\sin(g_1)]$$
$$A_2 = (g_1^2 + g_2^2)[g_1 \sinh(g_2)\cos(g_1) - g_2 \cosh(g_2)\sin(g_1)]$$
$$A_3 = -(g_1^2 + g_2^2)^2 \sinh(g_2)\sin(g_1)$$
$$A_4 = -g_1 g_2(g_1^2 + g_2^2)[g_2 \sinh(g_2)\cos(g_1) + g_1 \cosh(g_2)\sin(g_1)]$$
$$A_5 = 2g_1 g_2[1 - \cosh(g_2)\cos(g_1)] - (g_1^2 - g_2^2)\sinh(g_2)\sin(g_1)$$

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